

Improved photometric redshifts with color-constrained galaxy templates for future wide-area surveys

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ABSTRACT

Cosmology and galaxy evolution studies with LSST, *Euclid*, and WFIRST, will require accurate redshifts for the detected galaxies. In this study, we present improved photometric redshift estimates for galaxies using a template library that populates three-color space and is constrained by HST/CANDELS photometry. For the training sample, we use a sample of galaxies which have median photometric redshifts derived by combining the results of different redshift-fitting codes. It allows us to train on a large, unbiased galaxy sample having deep, unconfused photometry at optical-to-mid infrared wavelengths. Galaxies in the training sample are assigned to cubes in three-dimensional color space, $V - H$, $I - J$, and $z - H$. We then derive the best-fit spectral energy distributions (SEDs) of the training sample at the fixed CANDELS median photometric redshifts to construct the new template library for each individual color cube (i.e. color-cube-based template library). We derive photometric redshifts (photo- z) of our target galaxies in the GOODS-S and -N fields using our new color-cube-based template library. We find that our method yields σ_{NMAD} of 0.026 and an outlier fraction of 6% using only photometry in the LSST and *Euclid*/WFIRST bands. This is an improvement of $\sim 10\%$ on σ_{NMAD} and a reduction in outlier fraction of $\sim 13\%$ compared to other techniques. In particular, we improve the photo- z precision by about 30% at $2 < z < 3$. We also assess photo- z improvements by including K or mid-infrared bands to the *ugrizYJH* photometry. Our color-cube-based template library is a powerful tool to constrain photometric redshifts for future large surveys.

Keywords: galaxies: distances and redshifts

1. INTRODUCTION

Upcoming deep, wide-area surveys with *Euclid*, Wide-Field Infrared Survey Telescope (WFIRST), and the Large Synoptic Survey Telescope (LSST) will cover several thousands of square degrees of sky and observe billions of sources. Robust redshift estimates of such large numbers of galaxies are required both for galaxy and cluster evolution studies and three-dimensional weak lensing analysis. Ideally, spectroscopic redshifts yield the most accurate distances. However, spectroscopic surveys are biased towards strong emission line galaxies, are severely flux-limited, and biased against fainter galaxies. Obtaining spectroscopic redshifts of billions of objects is unfeasible, especially for faint galaxies that constitute the dominant fraction of the sample. Therefore, these future large surveys have to rely on photometric redshifts. This allows us to measure distances

to most galaxies with detections in multiple bands to very faint flux density levels with good statistical accuracy. More accurate and precise photo- z estimation is a challenging task, especially for the cosmological weak gravitational lensing measurements that have a stringent redshift bias requirement ($\Delta_z < 0.002$; Laureijs et al. (2011)).

Photometric redshifts are determined by developing and applying the mapping between redshift, observed flux densities and colors, using either template fitting (e.g. LePhare, EAZY, etc.) or machine-learning techniques. In recent years, many machine-learning approaches have been developed (Collister & Lahav 2004; Carrasco Kind & Brunner 2013, 2014; Rau et al. 2015; Hoyle 2016) and it has become popular because it is fast and powerful without the large systematic uncertainties associated with modeling dust and emission-line contributions (Speagle & Eisenstein 2017a). While machine-learning techniques are effective within their training sets (Sánchez et al. 2014; Masters et al. 2015; Hoyle et al. 2015; Speagle & Eisenstein 2017b; Fotopoulou & Paltani 2018; Bilicki et al. 2018), the mapping between various

colors (or flux densities) and redshift is not based on pre-existing physical knowledge, but is newly obtained every time, using a training sample with photometry and known redshifts. Therefore, machine-learning heavily depends on a training sample with spectroscopic redshifts, which are limited to brighter sources. Besides, Masters et al. (2015) showed spectroscopic redshifts have thus far, missed significant regions of color space which has motivated color-space targeted redshift surveys such as C3R2 (Masters et al. 2017).

In this study, we provide a new method to improve photometric redshift estimates using the template-fitting approach. While template-fitting approaches are fundamentally limited by the accuracy of existing templates and quality of the photometry, we implement a hybrid approach combining the methodology of both template fitting and empirical information obtained from a training sample. This relies on correlations between redshifts and observables such as colors, magnitude, and galaxy morphology. It is well-known that photometric tracers of strong features like the Lyman break at 912\AA , Balmer break at 4000\AA , and the $1.6\mu\text{m}$ bump provide redshift constraints (Sawicki (2002), Salvato et al. (2019)). It is also well-known that high redshift galaxies have fainter magnitudes and smaller sizes (e.g. van der Wel et al. 2014, and references therein) than the local universe. We aim to incorporate such information about the evolution of galaxy properties in multi-dimensional parameter space obtained from a training sample, to the template-fitting approach. Instead of the usual training set with spectroscopic redshifts, our training set consists of galaxies with known photometric redshifts derived as the average of multiple fitting techniques. The accuracy of photometric redshifts depends on the quality of photometry (high spatial resolution and high signal-to-noise) and wide spectral coverage with many photometric bands sampling the spectral energy distribution (SED; Brammer et al. 2008; Dahlen et al. 2010). For our purpose, the HST/Cosmic Assembly Near-infrared Extragalactic Legacy Survey (CANDELS; Grogin et al. 2011; Koekoer et al. 2011) is the optimal data set to train galaxy SED templates as it has excellent spectral coverage from the X-rays to far-infrared with exceptional depth and spatial resolution.

Here, we do not aim to invent new template models or a fitting code. We leverage HST/CANDELS photometry to constrain the range of templates and apply those to derive improved photometric redshifts for future wide area surveys. The structure of this paper as follows. CANDELS data is introduced in Section 2 together with selections of the training and target sample.

We describe the method constructing the new template library populated through three-dimensional color space and derive the photometric redshifts using our new template library in Section 3. In Section 4, we apply our method to the photometry expected from the LSST and *Euclid*-deep/WFIRST fields and validate the photometric redshift performance. We conclude in Section 5.

All magnitudes are expressed in the AB system unless stated otherwise. We use a standard Λ CDM cosmology with $H_0=70\text{ km s}^{-1}\text{ Mpc}^{-1}$, $\Omega_M=0.3$, and $\Omega_\Lambda=0.7$, which is broadly consistent with the recent results from *Planck*.

2. CANDELS DATA

We use *HST*/CANDELS (Grogin et al. 2011; Koekoer et al. 2011) observations that span five well-studied extragalactic fields: the Great Observatories Origins Survey (GOODS) Northern and Southern fields (GOODS-North and GOODS-South), the UKIRT Infrared Deep Sky Surveys (UKIDSS) Ultra Deep Field (UDS), the Extended Groth Strip (EGS) field, the Cosmic Evolution Survey (COSMOS) field. These fields together map out about 900 arcmin^2 of sky achieving 5σ point source limiting depth of ~ 27.3 AB mag (wide) and ~ 28.0 AB mag (deep) in the H (F160W) band. CANDELS multi-wavelength photometry catalogs span from the U-band through $8\mu\text{m}$ and are publicly available in each field (GOODS-S; Guo et al. (2013), GOODS-N; Barro et al. (2019), UDS; Galametz et al. (2013), EGS; Stefanon et al. (2017), COSMOS; Nayyeri et al. (2017)). Each field of CANDELS has been observed with many different telescopes and instruments in many different wavelengths. The multiple bands we use in this study for each field are listed below.

- GOODS-S (GS): 17 bands with U(Blanco/Mosaic II or VLT/VIMOS), F435W, F606W, F775W, F814W, F850LP from HST/ACS, F098M, F105W, F125W, F160W from HST/WFC3, Ks (VLT/ISAAC and HAWK-I), $3.6\mu\text{m}$, $4.5\mu\text{m}$, $5.8\mu\text{m}$, $8.0\mu\text{m}$ from Spitzer/IRAC.
- GOODS-N (GN): 16 bands with U(KPNO/Mosaic), F435W, F606W, F775W, F814W, F850LP from HST/ACS, F105W, F125W, F140W, F160W from HST/WFC3, Ks (Subaru/MOIRCS), K (CFHT/MegaCam), $3.6\mu\text{m}$, $4.5\mu\text{m}$, $5.8\mu\text{m}$, $8.0\mu\text{m}$ from Spitzer/IRAC.
- UDS: 17 bands with U (CFHT/MegaCam), B, V, R, i, z from Subaru/Suprime-Cam, F606W, F814W from HST/ACS, F125W, F160W from HST/WFC3, Y and Ks from VLT/HAWK-I, K

from UKIRT/WFCAM, $3.6\mu\text{m}$, $4.5\mu\text{m}$, $5.8\mu\text{m}$, $8.0\mu\text{m}$ from Spitzer/IRAC.

- EGS: 20 bands with u, g, r, i, z, from CFHT/MegaCam, F606w, F814W from HST/ACS, F125W, F140W, F160W from HST/WFC3, J1, J2, J3, H1, H2 from Mayall/NEWFIRM, Ks from CFHT/WIRCAM, $3.6\mu\text{m}$, $4.5\mu\text{m}$, $5.8\mu\text{m}$, $8.0\mu\text{m}$ from Spitzer/IRAC.
- COSMOS: 33 bands with u, g, r, i, z from CFHT/MegaPrime, B from Subaru/SuprimeCam, F606W, F814W from HST/ACS, F125W, F160W from HST/WFC3, Y and Ks from VISTA/VIRCAM, J1, J2, J3, H1, H2 from Mayall/NEWFIRM, $3.6\mu\text{m}$, $4.5\mu\text{m}$, $5.8\mu\text{m}$, $8.0\mu\text{m}$ from Spitzer/IRAC, and 12 intermediate bands in optical (IA484-IB827) from Subaru/SuprimeCam.

We correct the photometry in the catalogs for Galactic extinction using the value given by the IRSA Galactic dust reddening and extinction calculator¹. Extinction values are calculated at the center of each field based on Schlafly & Finkbeiner (2011) and are provided for a small set of filters. We then interpolate between the values to determine the extinction at the central wavelength of each filter in our data set. The median g-band extinction in the 5 fields is 0.040, 0.026, 0.061, 0.074, 0.027 mag for GN, GS, COSMOS, UDS and EGS, respectively. We note that applying the Galactic extinction correction slightly reduces the photometric redshift scatter by about 2 – 3% within the CANDELS fields covering $\sim 0.25\text{ deg}^2$.

It is worth noting that at depths beyond $\sim 25\text{ AB mag}$ in the optical, source confusion in seeing-limited imaging biases photometry (See Figure 1; e.g. Joint Survey Processing report 2019). The *Hubble*-quality spatial resolution of the deep optical data in the catalogs, combined with the priors on morphology and galaxy shapes applied for cataloging alleviates much of the confusion (Merlin et al. 2016). Thus, by using the CANDELS catalogs, we have the highest quality photometry available to build up the spectral energy distribution of the training sample of galaxies.

2.1. Training Sample

The galaxies in the training sample are required to have signal-to-noise (S/N) > 3 in U, B (F435W), F606W, F814W, F125W, F160W, K (or Ks), and $3.6\mu\text{m}$

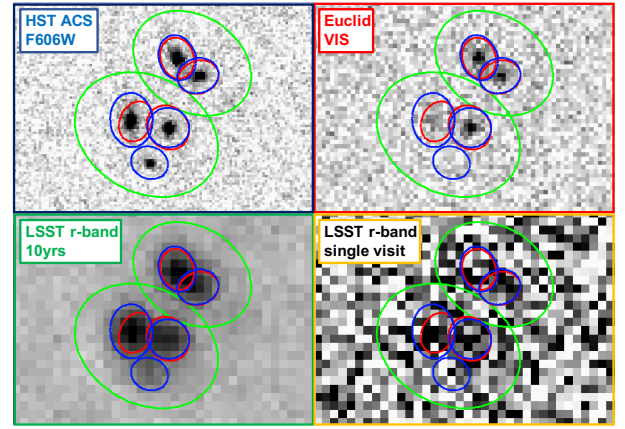


Figure 1. Source confusion in an optical band (centered at $\sim 6000\text{\AA}$), along with the isophotes derived from photometry on each of the images. The green isophotes are derived from ground-based resolution r-band data of 27.7 AB mag (e.g. LSST), the red isophotes are from the *Euclid*-quality optical data while the blue isophotes are from the HST/ACS data. The sources are barely detected in the LSST single epoch data which has a depth of 24.7 AB mag. In the absence of the deeper, space-resolution data, source confusion would result in both erroneous shape and photometry estimates in ground-based data and also affect catalog matching.

IRAC 1 channel to avoid having photometric uncertainties dominate redshift estimates. We then exclude suspicious sources and stars based on flags provided from the CANDELS photometry catalog;

- Suspicious sources: SExtractor PhotFlag = 0
- Stars: CLASS_STAR from SExtractor ≥ 0.98

SExtractor PhotFlag is used to designate suspicious sources that fall in regions of the image which are contaminated by stellar spikes or halos or edge effects (Guo et al. 2013). By using PhotFlag=0, we are able to exclude detections of star spikes, halos, and bright stars, as well as excluding sources that are either artifacts or falling at the edge of the image. We also exclude AGN-dominated sources identified from X-ray, IR, and/or radio detections which are about 1% of the sample. Since we do not have AGN templates to constrain their redshifts correctly, those sources usually result in higher uncertainties in the photometric redshifts compared to more normal galaxies (Salvato et al. 2009). This leaves us with a total of 39,391 galaxies as a training sample from all five CANDELS fields.

To build as large a training set as possible, which is not biased towards strong emission line sources, we consider only galaxies having photometric redshifts. For that, we use the CANDELS best-estimate photometric redshifts (photo-z), which is the median redshift from 11 different photometric redshift codes, each using dif-

¹ <https://irsa.ipac.caltech.edu/applications/DUST/>

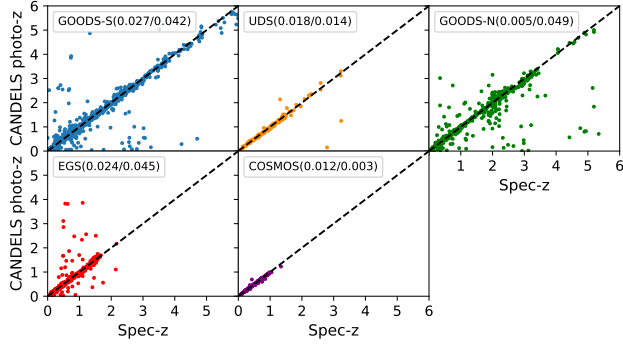


Figure 2. CANDELS median photometric redshifts (photo- z) vs. spectroscopic redshifts (spec- z) in five CANDELS fields. Galaxies are color-coded by fields; GOODS-S (blue), GOODS-N (green), UDS (orange), EGS (red), COSMOS (purple). The numbers in parenthesis are σ_{NMAD} and outlier fraction of each field. Galaxies with CANDELS spec- z have a different redshift range in each field. As one can see, most of galaxies at $z > 2$ are in the GOODS-S and -N fields. The photo- z scatter is small, particularly, in GOODS-N and COSMOS having σ_{NMAD} of 0.05 and 0.012, respectively. Also, all fields have less than 5% outliers indicating that these median photo- z s are a suitable training set for constraining galaxy SEDs.

ferent template spectral energy distributions, and priors (Dahlen et al. 2013).

The CANDELS median photo- z has yielded a photo- z precision of $\sigma_{NMAD} = 0.018$ and a catastrophic outlier of 4% at $0 < z < 6$ (see the definitions of statistics in Equation 1). The photo- z precision and outlier fraction of individual fields are presented in Figure 2 which shows the spectroscopic redshifts vs. photo- z in five fields with different colors. σ_{NMAD} of COSMOS and GOODS-N are small (0.012 and 0.005, respectively) because of an inclusion of medium-width optical bands (Laigle et al. 2019).

There are pros and cons of using a photo- z selected galaxy sample for developing the training set. Spectroscopically confirmed galaxies tend to be fewer in number, and biased towards bright, emission line galaxies. This can be problematic, especially, for future large photometric surveys that will image billions of galaxies including a large fraction of faint galaxies. Photo- z selected galaxies are an order of magnitude larger in number density, have less redshift accuracy but are relatively unbiased towards galaxies of a particular type. This alleviates the impact of spectroscopic misclassification, which tends to rely on single line redshifts. However, we are not immune from any systematic uncertainties inherent in CANDELS median photo- z s that introduce an additional scatter to our photo- z estimates.

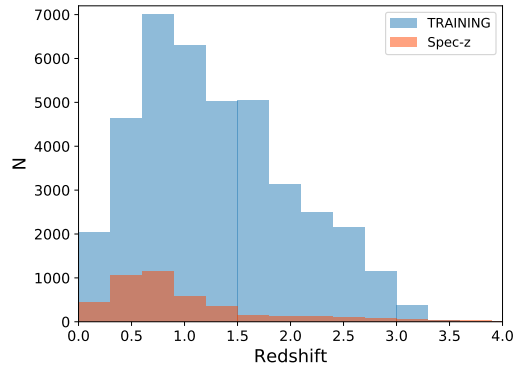
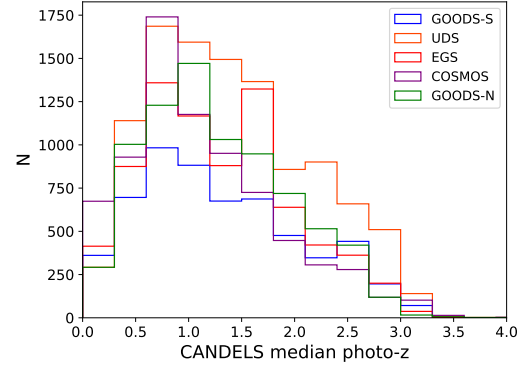


Figure 3. Top: Photo- z distributions of the training sample in five fields, GOODS-S (blue), GOODS-N (green), UDS (orange), EGS (red), COSMOS (purple). Bin size of the histogram is $\Delta z = 0.3$. The constraint of $S/N > 3$ in the U-band, restricts galaxies in the training sample to $z < 3.6$. Note that the red end of CTIO U-band in GOODS-S extends to 4120Å. Bottom: The photometric redshift distribution of the training sample with a blue histogram and the redshift distribution of galaxies having spec- z s with an orange histogram. We find that galaxies with $z_{spec} > 3.6$ are all in the GOODS-S and -N fields.

In Figure 3, we show the distribution of CANDELS median photo- z s of galaxies in the training sample. In the top panel, the training sample in different fields have photo- $z < 3.6$ because we require 3 sigma detection in U-band. The red end of the U-band filter in CANDELS extends to $\sim 4120\text{\AA}$, corresponding to the redshifted Lyman-break at $z \sim 3.6$.

2.2. Target Sample

We choose spectroscopically confirmed galaxies as the target sample to test the precision of photo- z derived from our work. We find spectroscopic redshifts (spec- z) from CANDELS catalogs and select galaxies having spec- z quality flag = 1 which means the spectroscopic data is secure. In total, we obtain 4,314 galaxies in five CANDELS fields after excluding suspicious sources,

Table 1. Comparison between HST/CANDELS and Future surveys

| Project | Wavelength range | 5σ sensitivity [AB mag] | Spatial Resolution [arcsec] |
|-------------------------|--------------------|--|-----------------------------|
| HST/CANDELS | VI(ACS) YJH (WFC3) | > 27.4 | 0.1(VI)/ 0.18(YJH) |
| LSST ^a | <i>ugrizy</i> | $< 24.7(\text{single}) / < 27.7(10 \text{ years})$ | 0.7 |
| Euclid NIR ^b | <i>YJH</i> | $24(\text{wide}) / 26(\text{deep})$ | 0.3 |
| WFIRST ^c | <i>zYJH+F184</i> | < 26.7 | 0.18 |

^a<https://docushare.lsst.org/docushare/dsweb/Get/LPM-17>

^b Euclid red book (Laureijs et al. 2011)

^c WFIRST AFTA 2015 Report (Spergel et al. 2015)

stars, and AGN candidates using the same flags mentioned in Section 2.1. In the bottom of Figure 3, we plot the spec- z distribution of those galaxies (orange histogram) with the photo- z distribution of the training sample (blue histogram). There is a relatively low fraction of high redshift galaxies beyond $z_{\text{spec}} = 2$, which is about 12% of the sample with spec- z , and most of them are in the GOODS-S and -N fields as can be seen in Figure 2.

Future large extragalactic surveys will observe several billions of galaxies and provide multi-wavelength catalogs consisting of optical (*ugrizy* from LSST) and near-infrared (*YJH* from *Euclid*², with the addition of F184 from WFIRST) photometry (See Table 1). The precision of photometric redshifts derived only using those 8 bands will be limited, since it will be challenging for ancillary data sets (e.g. narrow bands) to achieve the unprecedented sensitivities and areal coverage of these surveys. Since our primary goal is to assess the quality of redshifts that can be derived from these surveys, we select 1,696 galaxies in the GOODS-S and 1,355 galaxies in the GOODS-N field with spectroscopic redshifts as our target sample due to the multiwavelength photometry constraints. We only use photometry in 8 filters, including U from the ground-based telescopes, F435W, F606W, F814W, F850p from HST/ACS, and F105W, F125W, F160W from HST/WFC3, to derive the photo- z s. As shown in Table 1, the photometry used here can be considered as a case study for LSST-10 years+*Euclid*-deep/WFIRST-quality photometry albeit they are about a magnitude deeper, except U-band. We note that the spatial resolution of the data used here is far superior to that arising from LSST (0.1'' vs. 0.7'' FWHM), minimizing the impact of source confusion on photometric bias and thereby redshift errors.

In future work, we will apply the constraints from our training set to a completely independent spectroscopic redshift sample, such as that derived from the C3R2 survey; however those fields do not have consistently photometered data and deep near-infrared imaging such as that available in the CANDELS field.

3. METHOD: RECONSTRUCTING GALAXY TEMPLATES IN COLOR SPACE

Figure 4 illustrates four observed colors, $V-H$, $I-J$, $H-IRAC1$, and $z-H$, as a function of redshifts of galaxies (grey point) with the running median of colors for the training (blue) and target (orange) samples. IRAC1 corresponds to the *Spitzer*/IRAC 3.6 μm band. Most of the galaxies in the target sample have similar colors as the training sample, except one galaxy outside the $V-H$ range and three galaxies out of $H-IRAC1$ range. Clearly, there is a smooth and continuous redshift evolution through color space, implying that multicolor measurements can constrain redshift estimates. Each color varies with redshift in a different way, reaching a maximum at different redshifts. For example, the peak of the $I-J$ color is about $z = 1.6$ when the Balmer break (4000Å) falls between the I and J bands. For the same reason, $V-H$ peaks at $z = 1.1$, and $z-H$ peaks at $z = 1.9$. $H-IRAC1$ also shows a strong evolution with redshift, especially over $z < 1$, due to the 1.6 μm bump in galaxy SEDs arising from H- opacity (Sawicki 2002). However, single colors are degenerate in redshift as one can see from the scatter in the greyscales. We therefore use three color information to minimize degeneracies and use the multi-wavelength information to apply priors on galaxy SEDs.

3.1. Cubes in the three dimensional color space

In this study, we settled on using the following three colors after experimenting with various combinations of

² *Euclid* will also image in a broad optical band (VIS) from 550–900nm.

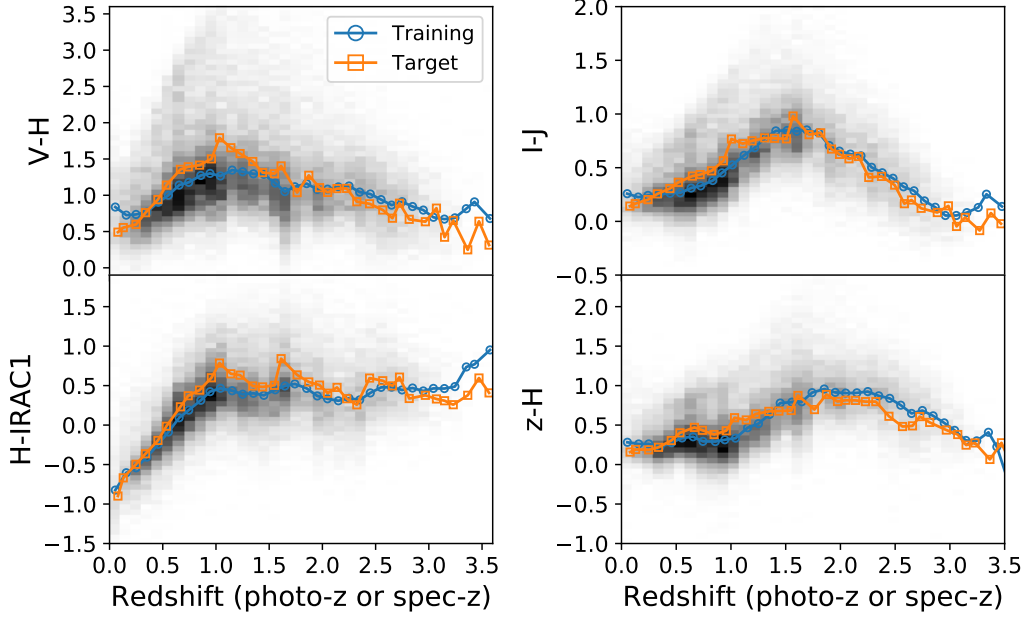


Figure 4. Observed colors in AB mag, $V - H$, $I - J$, $z - H$, $H - \text{IRAC1}$ vs redshift of the training and target sample. The greyscale is the number density of galaxies, i.e. the darker color represents a denser region. Blue and orange points represent the median colors at each redshift bin ($\Delta z = 0.1$) for the training and target sample, respectively. The distribution of observed colors of the target sample is similar to that of the training sample. There is a clear correlation between color and redshift with $V - H$, $I - J$, and $z - H$ peaking at about $z \sim 1.1$, 1.6 , and 1.9 , respectively, when the Balmer break falls between the two bands. $H - \text{IRAC1}$ evolves strongly at $z < 1$ due to the $1.6\mu\text{m}$ bump.

color space:

$$(V - H) = [-1.53, 5.07], \quad (I - J) = [-2.1, 3.5], \\ (z - H) = [-3.1, 6.9]$$

Although the relationship between color and redshift associated with photometric signatures is crucial, we find that the choice of colors strongly relies on the quality of photometry as well. For instance, even though $H - \text{IRAC1}$ is useful to constrain redshifts at $z < 1$, we find that $z - H$ improves photo- z estimates better than $H - \text{IRAC1}$ because the $3.6\mu\text{m}$ imaging is shallower than the z band in our dataset. Furthermore, in the template models, the band is contaminated by $3.3\mu\text{m}$ emission from polycyclic aromatic hydrocarbons and hot dust continuum whose intensity relative to the stellar population is rather poorly calibrated. We also tried $U - B$ color because this would be important for a secure detection of the Balmer break at low redshifts and for detecting the Lyman break at $z > 2$. However, U and B -bands in CANDELS are from ground-based telescopes (except F435W in the GOODS-S and -N fields), have shallower depth and lower resolution, so give weaker constraints on photo- z estimates.

Using $V - H$, $I - J$, and $z - H$ colors, we simply make bins in each color with a bin size of $\Delta = 0.2$ mag to build $0.2 \times 0.2 \times 0.2$ size cubes in three dimensional

color space as shown in the middle of Figure 5. As a result, all of the galaxies in the training sample are assigned to 1,286 cubes in $V - H$, $I - J$, and $z - H$ color space. The bin size is chosen because it is well matched to the photometric scatter of a typical $\sim 5\sigma$ source. In addition, using a larger bin size with less cubes results in heterogeneous templates within a cube. If on the other hand, one adopts more cubes with a smaller bin size, many cubes are not populated with templates from the training sample and the library of templates within each cube is sparsely sampled.

3.2. Best-fit SEDs of the training sample

We next need to obtain the best-fit spectral energy distribution (SED) of each galaxy in the training sample. This would then develop the library of templates applicable to each color cube in three-color space. We use the Flexible Stellar Population Synthesis (FSPS) models (Conroy et al. 2009; Conroy & Gunn 2010) and EAZY (Brammer et al. 2008) code for the fitting. EAZY has been widely used for large surveys, such as COSMOS (Muzzin et al. 2013), CANDELS (Dahlen et al. 2013), and 3D-HST (Skelton et al. 2014).

In order to obtain the best-fit SEDs for the training set of galaxies, we fix the redshift of each galaxy to its CANDELS median photo- z and fit FSPS templates to its

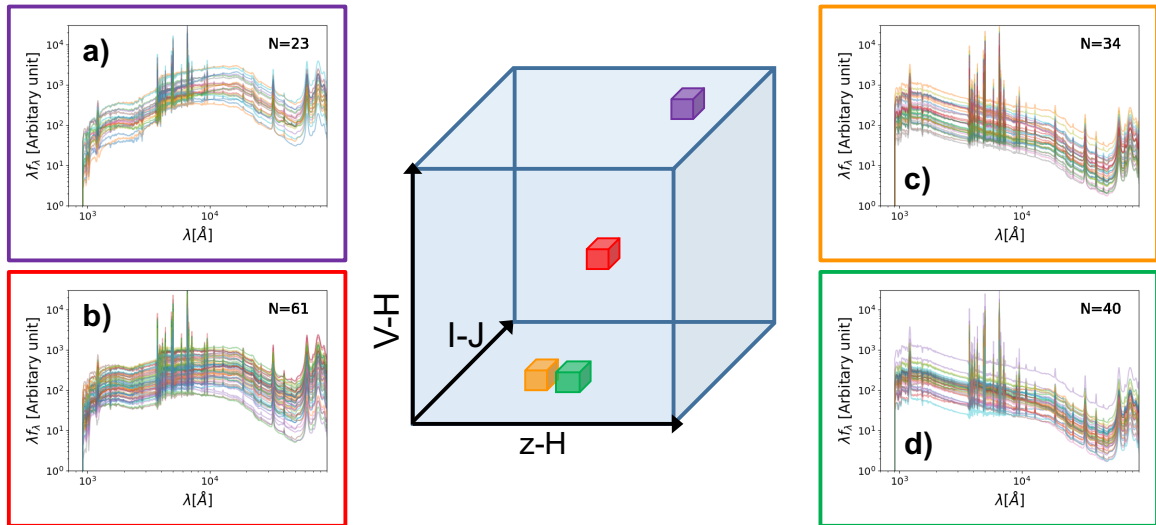


Figure 5. An illustration of the color-cube-based template library made with the training sample in three-dimensional color space. In the middle, there are four $0.2 \times 0.2 \times 0.2$ mag size cubes at different locations in the $V-H$, $I-J$, and $z-H$ color space. We also show the range of galaxy templates within the four color cubes with the number of rest-frame SEDs in the cube; a) for the purple cube, b) for the red, c) for the orange, and d) for the green cube. For example, 23 galaxies in the training sample are assigned to the purple cube so that the purple cube-based template (a) consists of 23 rest-frame SEDs (redshifted best-fit SEDs) of those galaxies. The red, green, and purple cubes have different types of galaxy SEDs. However, since the orange cube is located next to the green cube, their color-cube-based templates are very similar. If a target galaxy is located in the red cube, its photometric redshift is determined using the galaxy templates (61 galaxy SEDs in b) in the red box.

multiwavelength photometry. We apply the zero point offsets in Table A1 to the observed photometry of the training sample before doing the fits. It is well-known that adding zero-point offsets produce both lower scatter and outlier fraction of photometric redshifts (Dahlen et al. 2013). In Appendix A, we compute the zero point offsets for each individual filter and each field on the sample with spec-zs.

We use the linear combination of EAZY’s 12 galaxy templates derived from FSPS models with the method used for the original EAZY templates (Brammer et al. 2008) and trained on the UltraVISTA photometric catalogs (Muzzin et al. 2013). We use the default rest-frame template error function scaled by a factor of 0.2, which helps to account for systematic wavelength dependent uncertainties in the templates (see the details from the EAZY manual³).

Among 39,391 best-fit SEDs of the training sample, we exclude the 4.6% of fits having bad χ^2 to avoid using poor SED fits to the data. We then translate the best-fit SEDs to rest-frame wavelengths with CANDELS median photo-zs (hereafter, rest-frame SEDs).

3.3. Color-cube-based template library

We construct the color-cube-based template library using the rest-frame SEDs of the training sample. The

rest-frame SED of each galaxy in the training sample is assigned to its designated cube in $V-H$, $I-J$, and $z-H$ color space as illustrated in Figure 5. The color-cube-based template library is basically a collection of galaxy SEDs with similar three-color properties. By constraining the range of templates applicable to a galaxy using three-color space, we are constraining the range of star-formation histories possible, which helps alleviate color-redshift degeneracies. In Figure 5, as an example, we show four sets of galaxy templates of four cubes located differently in $V-H$, $I-J$, and $z-H$ space. Purple, blue, and green boxes have different types of galaxy SEDs representing mostly blue star-forming galaxies (green) or red passive galaxies (purple). However, if the cubes are located closely, their SEDs are very similar like green and orange cubes. We note that there are a maximum of 1109 rest-frame SEDs in a cube.

3.4. Photometric redshift estimates

There are two main parts in the algorithm for determining the photometric redshift of a target sample galaxy using the color-cube-based template library. The first is to find the correct color cube of the target galaxy in three dimensional color space. The second is to estimate the photo- z of the target galaxy by fitting its multiwavelength photometry with the range of color-cube-based templates with redshift as a free parameter. This is determined by minimizing the χ^2 between the ob-

³ http://www.astro.yale.edu/eazy/eazy_manual.pdf

served SED and the scaled color-cube-based templates within that cube.

The photo- z here is determined using the EAZY code modified to use the color-cube-based template library. As before, the photometry of the target galaxy is corrected by the zero-point offsets (Table A1). We do not allow linear combination of the library of galaxy templates as is done by EAZY. For example, if three galaxies are located in the purple cube in Figure 5, our method determines photo- z s of them only using the 23 galaxy templates in the purple box (plot a) and not a linear combination of subsets of the 23 templates. In other words, the rest-frame SEDs of the training sample of galaxies are assumed to be representative of, and applicable to the target sample of galaxies; the photo- z of the target is thereby derived from the best matching template. The photo- z of the target galaxy is the peak of the redshift probability distribution function (i.e. z_{peak} in EAZY terminology).

4. PHOTO-Z PERFORMANCE AND VALIDATION

We test the quality of photometric redshifts computed using the color-cube-based template library. We compare our photo- z estimates of the target sample of galaxies with their spectroscopic redshifts and to the photometric redshifts derived using EAZY on the same sample. Although CANDELS already has photo- z estimates using EAZY with the original templates from Brammer et al. (2008), we re-derive photo- z s using EAZY’s 12 FSPS galaxy templates with the same settings discussed in Section 3.2. We find that EAZY results in very similar or slightly better photo- z estimations compared to median CANDELS photo- z . Since the only difference between EAZY and our analysis is that we use a more constrained template library based on our training sample, we can validate the performance of our cube-based template library by comparing to the results from EAZY.

In this section, we investigate several implementations of our color-cube-based template library using only 8 bands (LSST+Euclid-deep/WFIRST-like) and an inclusion of longer wavelengths, Ks ($2.2 \mu m$) or IRAC 1 ($3.6 \mu m$).

4.1. Results with *ugrizYJH* photometry

The results of our photo- z fits are shown in Table 2 with statistics quantifying the accuracy of the photo- z estimates. Root mean square (RMS) of Δ_z , normalized median absolute deviation (NMAD) of Δ_z , and outlier

fraction (f_{out}) are defined in Equation 1.

$$\begin{aligned}\Delta_z &= (z_{spec} - z_{phot}) / (1 + z_{spec}) \\ \sigma_{full} &= \text{rms}[\Delta_z] \\ \sigma_{NMAD} &= 1.48 \times \text{median}[|\Delta_z|] \\ f_{out} &= N[|\Delta_z| > 0.15] / N_{obj}\end{aligned}\quad (1)$$

where, the outlier fraction, f_{out} is the fraction of outliers defined as objects with $|\Delta_z| > 0.15$. Figure 6 compares our photo- z estimates to spec- z s of galaxies in the two fields with σ_{NMAD} and f_{out} listed for each field. Note that the number of target galaxies are reduced to 1,655 and 1,343 in GOODS-S (GS) and -N (GN), respectively, since we are not able to obtain photo- z s of 2.4% galaxies in GS and 0.9% galaxies in GN because of two reasons. One, some of the target sample falls in cubes which do not have good templates because those cubes are only populated by SEDs with bad fits in the training sample. Two, we find that 1.6% of the target sample are not located in the training cubes; more than half of these are at $z_{spec} > 3.6$. The CANDELS spec- z sample has a surprising lack of high redshift galaxies with only 78 galaxies at $z_{spec} > 3.6$. This is rather inadequate to investigate the photo- z performance in this redshift range and we therefore do not present the results for galaxies not located in the training cubes. In the future, we will augment the target and training samples with the results of current spectroscopic surveys, particularly at $z_{spec} > 3$.

In Table 2, we compare σ_{full} , σ_{NMAD} , and f_{out} with EAZY results. We find that our method results in smaller σ_{full} , σ_{NMAD} , and f_{out} compared to EAZY, indicating the improvement of photo- z estimates. σ_{NMAD} of this study is 0.026 (0.028), while EAZY with 0.029 (0.028) in GS (GN). We improve the photo- z precision by about 10% in GS. In Figure 6, outliers residing below/above the red dashed lines which correspond to $|\Delta_z| = 0.15$ are visually reduced in this work, especially at $z < 3$. We find that f_{out} is about 6%, while EAZY has about 7-6% outliers. Outlier fraction is decreased by about 13% in GS compared to EAZY, while f_{out} in GN is almost identical between the two techniques. σ_{full} is 0.194 and 0.157 in GS and GN, similar to EAZY. We also plot σ_{full} , σ_{NMAD} , and f_{out} as a function of redshift in Figure 7. At all redshifts considered here, photo- z precision is relatively high with $0.02 < \sigma_{NMAD} < 0.04$. In particular, at $z > 2$, σ_{NMAD} of our method is significantly small, $\sigma_{NMAD} \sim 0.03 - 0.04$, comparing to EAZY with ~ 0.06 at $2 < z < 3$, indicating that we can improve the photo- z precision by about 30% by using the color-cube-based template library at $z > 2$. Other statistics, σ_{full} and f_{out} are smaller than ones from EAZY at

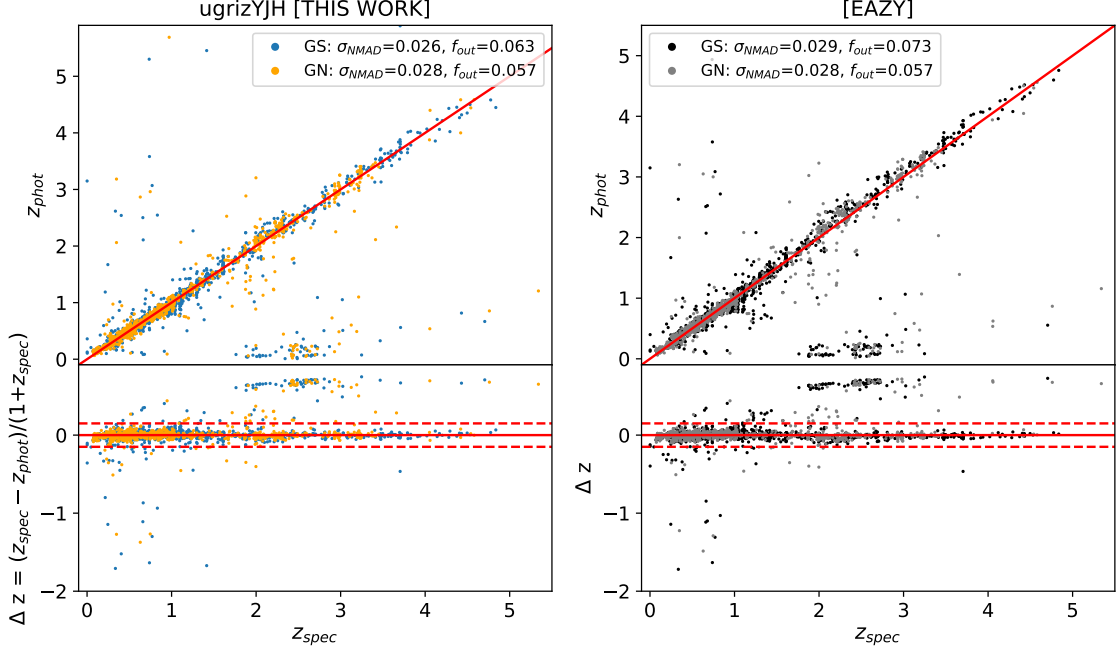


Figure 6. Comparison between spectroscopic redshifts (z_{spec}) and photo- z estimates (z_{phot}) of the target sample in GOODS-S (GS) and GOODS-N (GN). Left: A result from our method with GOODS-S (blue) and GOODS-N (orange) galaxies. Right: The result from EAZY for the same galaxies (GS: black, GN: grey). We plot z_{phot} vs z_{spec} in the top panel and $\Delta_z (= (z_{\text{spec}} - z_{\text{phot}})/(1 + z_{\text{spec}}))$ vs. z_{spec} in the bottom panel with σ_{NMAD} and the outlier fraction f_{out} measured for each field. $\Delta_z = 0$ is shown as a red line with dashed lines indicating $|\Delta_z| = 0.15$. Galaxies located below/above the dashed lines are classified as outliers. Here, we use LSST+*Euclid*-deep/WFIRST-quality multi-band photometry to derive photo- z using the color-cube-based template library made from $V - H$, $I - J$, and $z - H$ colors. Our method results in an improved photo- z precision of $\sigma_{\text{NMAD}} = 0.26 - 0.28$ and smaller outlier fraction of about 6% comparing to EAZY with $\sigma_{\text{NMAD}} = 0.028 - 0.029$ and $f_{\text{out}} = 6\text{-}7\%$.

Table 2. Photometric redshift results with LSST+*Euclid*-deep/WFIRST-quality photometry (*ugrizYJH* bands).

| Field Name | N | THIS WORK | | | EAZY | | |
|------------|------|------------------------|------------------------|------------------|------------------------|------------------------|------------------|
| | | σ_{full} | σ_{NMAD} | f_{out} | σ_{full} | σ_{NMAD} | f_{out} |
| GS | 1655 | 0.194 | 0.026 | 0.063 | 0.194 | 0.029 | 0.073 |
| GN | 1343 | 0.157 | 0.028 | 0.056 | 0.161 | 0.028 | 0.056 |

$1.5 < z < 3.5$, while similar to EAZY at lower redshifts and the highest redshift bin ($z > 3.5$). Overall, our result shows that the color-cube-based template library trained with CANDELS data in color space is powerful for estimating photometric redshifts, especially when the number of photometric bands is limited.

4.2. Results with an inclusion of IRAC 1 or Ks bands

It has been shown that the derivation of photometric redshifts benefit from having multiple narrow bands and having broad wavelength coverage (Dahlen et al. 2010), especially if they straddle strong features in galaxy

SEDs such as the Balmer break and $1.6\mu\text{m}$ bump. It is therefore worth assessing if the addition of longer wavelength bands to the planned *ugrizYJH* photometry would be beneficial for photo- z derivation. We investigate here the redshift accuracy when we add longer near-infrared bands, Ks ($2.2\mu\text{m}$) or IRAC 1 ($3.6\mu\text{m}$) to the *ugrizYJH* bands.

We first derive photo- z s using our method by including the IRAC 1 band, i.e. 9 bands adding IRAC 1 to the *ugrizYJH* bands. The results are shown in Table 3. In comparison to the result only using *ugrizYJH* (Table 2), we are able to improve the overall performance

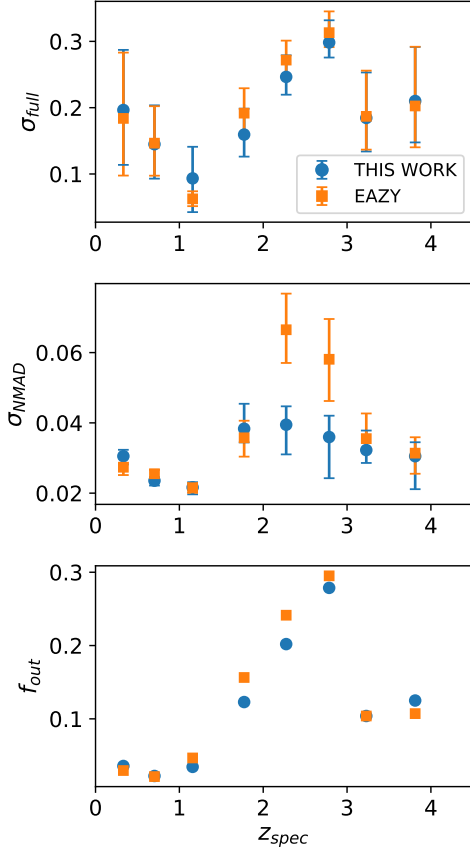


Figure 7. Three photo-z statistics, σ_{full} , σ_{NMAD} , and f_{out} as a function of spec-z (z_{spec}). Bin size is $\Delta z_{spec} = 0.5$, while the last bin include galaxies at $3.5 < z_{spec} < 5.5$. Statistics measured in each redshift bin are shown on the Y-axis for this work (blue) and EAZY (orange). X-axis is the median z_{spec} in each bin. Error bars of σ_{full} and σ_{NMAD} are determined by bootstrapping the sample. Our photo-z performance at $1.5 < z < 3.5$ is better with smaller values of σ_{full} , σ_{NMAD} , and f_{out} than ones from EAZY, while similar to EAZY at $z < 1.5$ and $z > 3.5$. In particular, our method has a very high precision of $\sigma_{NMAD} < 0.04$ at $z > 2$ comparing to EAZY with < 0.06 , indicating about 30% improved performance at $z > 2$.

of photo-zs when an IRAC 1 band is included. We obtain a smaller $\sigma_{NMAD} = 0.025$ in both GS and GN. The fraction of outliers decreases from 6.3% to 5.1% for the GS and 5.6% to 5.2% for the GN. Likewise, we find σ_{full} drops dramatically in both fields, to 0.141 (GS) and 0.130 (GN), which corresponds to about a 27% and 17% decrease, respectively. This implies that the inclusion of the IRAC 1 band has a significant impact on reducing outliers. This is particularly relevant for the 40 deg^2 *Euclid*-deep fields which are the target of *Spitzer* observations achieving depths of $5\sigma=24.6$ AB mag (PI: P. Capak).

Table 3. Photo-z precision when including IRAC1 or Ks bands

| | Field name | σ_{full} | σ_{NMAD} | f_{out} |
|-------------|------------|-----------------|-----------------|-----------|
| with IRAC 1 | GS | 0.141 | 0.025 | 0.051 |
| | GN | 0.130 | 0.025 | 0.052 |
| with K band | GS | 0.168 | 0.029 | 0.063 |
| | GN | 0.158 | 0.027 | 0.060 |

Next, we derive photo-zs with 9 bands adding Ks band to the ugrizYJH. Ks band photometry in GS is quite deep with VLT/HAWK-I (5σ depth of 26.45 AB), while the GN Ks-band is shallower, ~ 24.4 AB mag. As shown in Table 3, the inclusion of the shallow Ks band photometry has only a marginal effect for galaxies in GN when compared to the result without Ks-band. The relatively low impact on the photometric redshifts when including Ks-band could be attributed to the fact that the H band from HST already provides a tight constraint on the SED longwards of the Balmer break and the $1.6\mu\text{m}$ bump has a relatively small effect on photo-z estimation at the redshift range of interest. Despite having deeper Ks band in GS, we obtain worse results with a photo-z precision of $\sigma_{NMAD} = 0.29$. The origin of this discrepancy is a puzzle and might require a careful re-evaluation of the HAWK-I photometry.

5. CONCLUSION: AN INTRODUCTION OF NEW TEMPLATE LIBRARIES

We investigate a new method for improving the accuracy of photo-z estimates for future wide area surveys which will be deep and have limited multi-wavelength coverage. There are two unique features in our method. To have a large unbiased galaxy sample, we select the training sample of galaxies to have CANDELS photometric redshifts which are a median of 11 different photo-z codes. We then populate three-dimensional color space, $V-H$, $I-J$, and $z-H$ with the best fit SEDs for this training sample to derive a color-cube-based template library trained with high-quality multi-wavelength CANDELS photometry.

We apply our color-cube-based template library to a target sample of galaxies with ugrizYJH photometry (LSST+ *Euclid*/WFIRST-like) and derive their photo-zs. We quantify the performance of our method for determining photo-zs by comparing with spec-zs. We derive a photo-z precision of $\sigma_{NMAD} = 0.026 - 0.028$ and outlier fraction of about 6% for galaxies in the GOODS-S and -N fields. We compare our results with EAZY using their templates and find that our method improves the photo-z precision by about 10% and reduces the outliers by about 13%. In particular, the color-cube-based template library is very effective at $2 < z < 3$ with sig-

nificantly small σ_{NMAD} of ~ 0.04 , comparing to EAZY which yields $\sigma_{NMAD} \sim 0.06$. We also investigate the case of including additional longer wavelength photometry, e.g. Ks-band or IRAC1 ($3.6\ \mu\text{m}$), to the ugrizYJH photometry. Including deep IRAC1 photometry results in smaller $\sigma_{NMAD} = 0.025$ and has the most significant impact on reducing outliers. However, the inclusion of Ks-band has marginal to no effect on the photometric redshifts mainly due to the fact that the deeper H-band data from *HST* already provides a tight constraint on the galaxy SEDs.

Overall, our result clearly shows that the color-cube-based template library trained with CANDELS data in multi-dimensional color space is a powerful tool for determining photometric redshifts using the template-fitting approach, particularly for future large extragalac-

tic surveys which will provide deep ugrizYJH photometry. These results will be highly applicable to the LSST and *Euclid*-deep fields which span $40\ \text{deg}^2$ and span similar depths as those sampled in this study. They can also be applied to the LSST+WFIRST high latitude survey sky area which will span $2200\ \text{deg}^2$.

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APPENDIX

A. ZERO POINT OFFSETS

As described in Skelton et al. (2014) and Dahlen et al. (2013), the application of offsets to the photometry can improve photometric redshift fits. These are called zero point offsets but in general, since the instrument photometry is calibrated with high accuracy, is unlikely to be due to actual errors in zero points. It is therefore thought that any offsets are likely due to the mismatch between the SED templates and real galaxy SEDs (Guo et al. 2013) although inconsistent isophotal apertures across bands may also be a contributing factor. Zero point offsets are determined for individual filters and vary in different fields. We measure the zero point offset of each band by fitting the EAZY FSPS templates to the observed photometry of galaxies with spectroscopic redshifts as in Skelton et al. (2014). The offsets are computed iteratively by minimizing the median differences between the measured photometry and best fit templates integrated over the bandpasses in a particular field. This correction is then applied to the photometry of all sources. In principle, the correction should be calculated on a spec-z sample which is distinct from the sample on which the quality of photo-z is measured. However, due to the limited number of spec-zs available, these corrections are applied on the same sample. With future spectroscopic surveys, this is likely to be improved.

The listed zero point offset corrections in Table A1 have been applied to the photometry when we obtain the best-fitted SED of each galaxy in the training sample in Section 3.2 as well as when we derive the photometric redshifts of the target sample using the color-cube-based template library in Section 3.4. The values in the table are multiplicative numbers to be applied to the flux densities in the corresponding filters.

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Table A1. Zero point offsets for individual filters in 5 fields

| GOODS-S | | GOODS-N | | UDS | | EGS | | COSMOS | |
|---------------|-----------|-----------|-----------|---------------|-----------|------------|-----------|--------------|-----------|
| Filter | zp offset | Filter | zp offset | Filter | zp offset | Filter | zp offset | Filter | zp offset |
| U Blanco | 1.0404 | U KPNO | 0.9483 | U CFHT | 1.1115 | u CFHT | 0.9980 | u CFHT | 0.9070 |
| U VLT | 1.0304 | F435W | 1.0616 | B Subaru | 0.9999 | g | 0.9619 | g | 0.9114 |
| F435W | 0.9855 | F606W | 0.9928 | V | 0.9902 | r | 0.9960 | r | 0.9506 |
| F606W | 0.9929 | F775W | 0.9811 | R | 0.9144 | i | 0.9786 | i | 0.9415 |
| F775W | 0.9892 | F814W | 0.9826 | i | 0.9878 | z | 0.9763 | z | 0.9503 |
| F814W | 0.9910 | F850LP | 0.9531 | z | 0.9960 | F606W | 0.9315 | B Subaru | 0.9888 |
| F850LP | 0.9674 | F105W | 0.9900 | F606W | 1.0518 | F814W | 0.9624 | F606W | 0.9476 |
| F098M | 1.0210 | F125W | 1.0020 | F814W | 0.9834 | F125W | 1.0025 | F814W | 0.9784 |
| F105W | 1.0053 | F140W | 0.9915 | Y HAWK-I | 1.0144 | F140W | 0.9907 | F125W | 1.0183 |
| F125W | 1.0070 | F160W | 1.0000 | F125W | 1.0065 | F160W | 1.0000 | F160W | 1.0000 |
| F160W | 1.0000 | Ks Subaru | 0.9540 | F160W | 1.0000 | J1 NEWFIRM | 0.9255 | Y UVISTA | 0.9012 |
| Ks VLT/ISAAC | 1.0675 | Ks CFHT | 0.9373 | Ks VLT/HAWK-I | 1.0016 | J2 | 0.9117 | Ks UVIST | 0.9202 |
| Ks VLT/HAWK-I | 1.0100 | IRAC 1 | 1.0070 | K UKIRT/WFCAM | 1.0039 | J3 | 0.9248 | J1 NEWFIRM | 0.8942 |
| IRAC 1 | 0.9616 | IRAC 2 | 0.9871 | IRAC 1 | 1.0006 | H1 | 0.9060 | J2 | 0.9034 |
| IRAC 2 | 1.0175 | IRAC 3 | 1.0404 | IRAC 2 | 0.9825 | H2 | 0.9316 | J3 | 0.9223 |
| IRAC 3 | 0.9940 | IRAC 4 | 1.0507 | IRAC 3 | 0.9447 | Ks WIRCAM | 0.9636 | H1 | 0.8969 |
| IRAC 4 | 1.0116 | | | IRAC 4 | 0.9848 | IRAC 1 | 0.9857 | H2 | 0.8777 |
| | | | | | | IRAC 2 | 0.9783 | IRAC 1 | 0.9372 |
| | | | | | | IRAC 3 | 1.0594 | IRAC 2 | 0.9166 |
| | | | | | | IRAC 4 | 0.7614 | IRAC 3 | – |
| | | | | | | | | IRAC 4 | – |
| | | | | | | | | IA484 Subaru | 0.9441 |
| | | | | | | | | IA527 | 0.9582 |
| | | | | | | | | IA624 | 0.9482 |
| | | | | | | | | IA679 | 0.7946 |
| | | | | | | | | IA738 | 0.9942 |
| | | | | | | | | IA767 | 0.9608 |
| | | | | | | | | IA427 | 0.9452 |
| | | | | | | | | IA464 | 0.9458 |
| | | | | | | | | IB505 | 0.9414 |
| | | | | | | | | IB574 | 0.9749 |
| | | | | | | | | IB709 | 0.9618 |
| | | | | | | | | IB827 | 0.9809 |

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